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DIFFERENTIATION OF F(X) = LOG(X).

BY PROF. JAMES G. CLARK, LIBERTY, MO.

THE characteristic property of this function is expressed by the equation

$$F(x^m) = mF(x),$$

from which we have at once

$$dF(x^m) = mdF(x).$$

Let us assume $dF(x) = f(x)dx$, in which $f(x)$ is an unknown function. We have then

$$dF(x^m) = f(x^m)dx^m = mx^{m-1}f(x^m)dx.$$

Therefore by substitution,

$$mx^{m-1}f(x^m)dx = mf(x)dx, \text{ and}$$

$$f(x^m) = \frac{f(x)}{x^{m-1}}, \text{ or } x^m f(x^m) = xf(x).$$

This relation being independent of the value of m, it follows that

$$xf(x) = M, \text{ a constant.}$$

Therefore $f(x) = \frac{M}{x}$, and $dF(x) = d\log(x) = \frac{M}{x}dx$.

In this equation M is of unknown value. Its value cannot be directly determined from the general properties of logarithms, but it may be observ'd that in whatever system the logarithms be taken, $\log x \div M$ will be the logarithm of x in some other system, whose base may be denoted by e. W have, therefore, from the equation

$$d\log(x) = \frac{M}{x}dx,$$

$$d\left\{\frac{\log x}{M}\right\} = d\log_e x = \frac{dx}{x}.$$

There is therefore one system for which the multiplier of $dx \div x$ is unity, and it readily follows that the multipliers of $dx \div x$ in different systems are proportional to their moduli. It may be shown in the usual way by Maclaurin's theorem that e = Napierian base.

SOLUTIONS OF PROBLEMS IN NUMBER ONE.

SOLUTIONS of problems in No. 1 have been received as follows:

From Marcus Baker, 423, 426; Florian Cajori, 423, 424, 425, 426, 427; George E. Curtis, 423; Prof. W. P. Casey, 423, 426; G. W. Evans, 423;

Prof. A. B. Evans, 427; Prof. Wm. Hoover, 423, 425, 426; H. Heaton, 423, 424, 425, 426, 427; Prof. H. P. Manning, 426, 427; Prof. J. W. Nicholson, 426, 427; A. F. Parsons, 423, 424, 426; Prof. P. H. Philbrick, 423, 424, 425, 426, 427; P. Richardson, 423, 426; Prof. J. Scheffer, 423, 426, 427; Prof. E. B. Seitz, 423, 426, 427; R. S. Woodward, 423, 424, 425, 426, 427.

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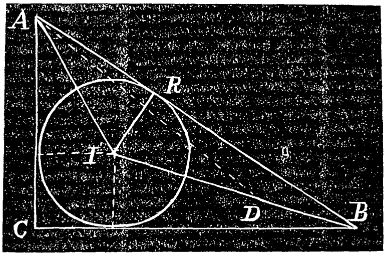
423. *By E. Millwee, Add-Ran College, Granbury, Texas.*—"Given the hypothenuse of a right-angled triangle and the difference of the two lines drawn from the acute angles to the centre of the insribed circle, to find the triangle."

SOLUTION BY A. F. PARSONS, ADELAIDA, CAL.



Let ABC be the triangle, AI, BI the bisecting lines of which the difference is given. Draw IR perpendicular to AB.

Put $AB=c$, and the given difference $=d$. Also let $r=IR$, $x=AI$ and $y=AR$. Then $y+r=AC$, $c-y+r=BC$, $x+d=IB$, and $c-y=RB$.

By right triangles we have

$$x^2=y^2+r^2, \qquad (1)$$

$$(x+d)^2-r^2=(c-y)^2, \text{ or } x^2+2xd+d^2-r^2=c^2-2cy+y^2, \qquad (2)$$

$$(y+r)^2+(c-y+r)^2=c^2, \text{ or } y^2-cy=-(r^2+cr). \qquad (3)$$

Substitute the values of x^2 and x from (1) in (2) and we get

$$d\sqrt{(y^2+r^2)}=\tfrac{1}{2}(c^2-d^2)-cy. \qquad (4)$$

From (3) and (4) we get a quadratic whose root is

$$r=\tfrac{1}{2}(c^2-d^2)(\sqrt{2}-1). \qquad (5)$$

Substituting values of r and r^2 from (5) in (3) we get the quadratic,

$$y^2-cy=\tfrac{1}{4}(c^2-d^2)[(d^2\div c^2)(3-2\sqrt{2})-1]; \qquad (6)$$

$$\therefore y=\tfrac{1}{2}\{c\pm d\sqrt{[\tfrac{1}{2}(c^2-d^2)(3-2\sqrt{2})+1]}\}. \qquad (7)$$

and the sides are

$$\frac{c^2+(c^2-d^2)(\sqrt{2}-1)}{2c}\pm\frac{d}{2}\sqrt{\left[\frac{c^2-d^2}{c^3}(3-2\sqrt{2})+1\right]}. \qquad (8)$$

Or, take $ID=IA$ and connect AD. Then $DB=d$, $\angle AID=135°$, $\angle IDA=22\tfrac{1}{2}°$ and $\angle BDA=157\tfrac{1}{2}°$; $\therefore DAB=\sin^{-1}[(d\div 2c)(2-\sqrt{2})]$. Therefore DBA and consequently ABC become known. Then

$$AC=c\sin ABC; \qquad BC=c\cos ABC.$$

SOLUTION BY P. RICHARDSON, BROOKLYN, N. Y.

Let $2x$ and $2y$ be the sides, r = the radius of the inscribed circle, $2h$ = the hypothenuse, and $2d$ = the given difference; then the following equations can be easily found;

$$x+y-r=h\ldots(1),\quad 2xy=r(x+y+h), \qquad (2)$$

$$x^2+y^2=h^2\,..\,(3),\quad \sqrt{[(2x-r)^2+r^2]}-\sqrt{[(2y-r)^2+r^2]}=\pm d. \qquad (4)$$

In (4) put value of r from (1), perform the operations indicated under the radical signs and substitute for (x^2+y^2) its value from (3), reduce and transpose second radical, and there results

$$\sqrt{(h^2-hy)}=\pm d+\sqrt{(h^2-hx)}.$$

Square this equation, transpose and substitute h^2 for x^2+y^2 and we have

$$h^4-4h^2d^2+d^4=2h^2xy-2hd^2(x+y). \qquad (5)$$

From (1) and (2) we get $x+y=r+h$ and $2xy=r(r+2h)$, put these values in (5), transpose and we have he quadratic equation

$$h^2r^2+(2h^3-2hd^2)r=h^4-2h^2d^2+d^4, \text{ whence}$$

$$r=(\pm\sqrt{2}-1)(h^2-d^2)\div h; \qquad (6)$$

$$\therefore\ 2x=r+h+\sqrt{(h^2-r^2-2rh)},\qquad 2y=r+h-\sqrt{(h^2-r^2-2rh)}.$$

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424 *By Prof. De Volson Wood.*—"Required the equation to the locus which is at a constant, internal, normal dist. from the four cusp epicycloid."

SOLUTION BY R. S. WOODWARD, C. E., DETROIT, MICHIGAN.

The equations of this epicycloid are

$$\left.\begin{aligned} x &= \tfrac{1}{4}r(5\cos\theta-\cos 5\theta)\\ y &= \tfrac{1}{4}r(5\sin\theta-\sin 5\theta)\end{aligned}\right\}, \qquad (1)$$

r being the radius of the fixed circle. The equation to the normal to this curve is

$$\eta-y=-\frac{dx}{dy}(\varepsilon-x),$$

and the distance of (ε,η) from (x,y), which in this problem is constant, is defined by

$$(\eta-y)^2+(\varepsilon-x)^2=\left(1+\frac{dx^2}{dy^2}\right)(\varepsilon-x)^2=\left(1+\frac{dy^2}{dx^2}\right)(\eta-y)^2=c^2 \text{ say.} \qquad (2)$$

Since by (1) $\dfrac{dx}{dy}=\dfrac{\sin\theta-\sin 5\theta}{\cos\theta-\cos 5\theta}=\cot 3\theta$, (2) gives

$$\begin{aligned}\varepsilon &= x\mp c\sin 3\theta=\tfrac{1}{4}r(5\cos\theta-\cos 5\theta)\mp c\sin 3\theta,\\ \eta &= y\pm c\cos 3\theta=\tfrac{1}{4}r(5\sin\theta-\sin 5\theta)\pm c\cos 3\theta,\end{aligned}$$

which define the locus.

425. "A cylinder rolls down the upper plane of a wedge, the wedge being upon a perfectly smooth horizontal plane; required the velocity of the cylinder when it shall have rolled a distance l on the plane."

SOLUTION BY WILLIAM HOOVER, A. M., DAYTON, OHIO.

Let m and m' be the masses of the cylinder and wedge, a the radius of the cylinder, mk^2 its moment of inertia, α the inclination of the upper face of wedge to the horizon, θ the angle through which the cylinder has rolled in any time t from the beginning of motion, F and R the reactions of the cylinder and inclined plane, parallel and perpendicular to the upper surface of the wedge. Let x' be the distance the wedge is at any time from an assum'd point in the horizontal plane, x and y the coordinates of the center of gravity C, of the cylinder, the axis of which is all the time supposed horizontal.

For the cylinder, resolving horizontally and vertically and taking moments about the center of gravity,

$$m\frac{d^2x}{dt^2} = -R\sin\alpha + F\cos\alpha, \tag{1}$$

$$m\frac{d^2y}{dt^2} = R\cos\alpha + F\sin\alpha - mg, \tag{2}$$

$$mk^2\frac{d^2\theta}{dt^2} = Fa. \tag{3}$$

For the single motion of the wedge,

$$m'\frac{d^2x'}{dt^2} = R\sin\alpha - F\cos\alpha. \tag{4}$$

We also have

$$y\cos\alpha = a + (x - x')\sin\alpha. \tag{5}$$

From (1) and (4),

$$m\frac{d^2x}{dt^2} + m'\frac{d^2x'}{dt^2} = 0. \tag{6}$$

From (5)

$$\cos\alpha\frac{d^2y}{dt^2} = \sin\alpha\left(\frac{d^2x}{dt^2} - \frac{d^2x'}{dt^2}\right), \tag{7}$$

which with (6) gives

$$m'\cos\alpha\frac{d^2y}{dt^2} = (m+m')\sin\alpha\frac{d^2x}{dt^2}. \tag{8}$$

Since the cylinder rolls,

$$\frac{dx}{dt} = \frac{dx'}{dt} - a\cos\alpha\frac{d\theta}{dt}, \text{ whence}$$

$$a\cos\alpha\frac{d^2\theta}{dt^2} = \frac{d^2x'}{dt^2} - \frac{d^2x}{dt^2}. \tag{9}$$

From (6) and (9), $m'a\cos\alpha\frac{d^2\theta}{dt^2} = -(m+m')\frac{d^2x}{dt^2}.$ (10)

Eliminating R and F from (1), (2) and (3),

$$a\cos\alpha\frac{d^2x}{dt^2}+a\sin\alpha\frac{d^2y}{dt^2} = k^2\frac{d^2\theta}{dt^2}-ag\sin\alpha. \quad (11)$$

Substituting (8) and (10) in (11) gives

$$[m'a^2\cos^2\alpha+(m+m')(a^2\sin^2\alpha+k^2)]\frac{d^2x}{dt^2} = -m'a^2g\sin\alpha\cos\alpha. \quad (12)$$

Substituting in (8) gives

$$[m'a^2\cos^2\alpha+(m+m')(a^2\sin^2\alpha+k^2)]\frac{d^2y}{dt^2} = -(m+m')a^2g\sin\alpha. \quad (13)$$

Let $x=b$, $y=c$, when $t=0$. Then integrating (12) and (13) twice,

$$x-b = -m'a^2gt^2\sin 2\alpha\div 4[m'a^2\cos^2\alpha+(m+m')(a^2\sin^2\alpha+k^2)], \quad (14)$$

$$y-c = -(m+m')a^2gt^2\sin^2\alpha\div 2[m'a^2\cos^2\alpha+(m+m')(a^2\sin^2\alpha+k^2)] \quad (15)$$

(14)÷(15) gives $(m+m')\sin\alpha\,(x-b) = m'\cos\alpha\,(y-c)$ (16), the equation of a straight line, the path described in space by the center of gravity of the cylinder.

Multiplying (12) by $2dx\div dt$ and integrating,

$$\frac{dx}{dt} = \sqrt{\left\{\frac{m'a^2g\sin 2\alpha\,(b-x)}{m'a^2\cos^2\alpha+(m+m')(a^2\sin^2\alpha+k^2)}\right\}},$$

the horizontal velocity of the center of gravity of the cylinder for any abscissa x. Let $s=$ the distance moved over by the wedge while the cylinder moves over the distance l; then $(m+m')s = ml\cos\alpha$; hence when the cylinder has moved the distance l, the value of x is

$$b-\left(l\cos\alpha-\frac{ml\cos\alpha}{m+m'}\right) = b-\frac{m'l\cos\alpha}{m+m'}, \text{ which gives}$$

$$\frac{dx}{dt} = \sqrt{\left\{\frac{m'^2a^2gl\sin 2\alpha\cos\alpha}{(m+m')[m'a^2\cos^2\alpha+(m+m')(a^2\sin^2\alpha+k^2)]}\right\}},$$

the horizontal velocity of C when the cylinder has rolled the distance l on the upper surface of the wedge.

From (16) we have

$$\frac{\sqrt{(dx^2+dy^2)}}{dy} = \frac{\sqrt{[(m+m')^2\sin^2\alpha+m'^2\cos^2\alpha]}}{(m+m')\sin\alpha},$$

the cos. of the angle which the locus of C makes with the axis of abscissas.

Therefore, resolving the above velocity in the direction of the locus, the required velocity is

$$\frac{m'a}{(m+m')\sin\alpha}\sqrt{\frac{gl\sin 2\alpha\cos\alpha[(m+m')^2\sin^2\alpha+m'^2\cos^2\alpha]}{(m+m')[m'a^2\cos^2\alpha+(m+m')(a^2\sin^2\alpha+k^2)]}}.$$

The velocity of the cylinder will be less than if it rolled down the same distance on a fixed inclined plane.

426. *By William Hoover, A. M.*—"Eliminate θ from the equations

$$(a+b)\tan(\theta-\varphi)=(a-b)\tan(\theta+\varphi), \qquad (1)$$

$$a\cos 2\varphi+b\cos 2\theta=c." \qquad (2)$$

SOLUTION BY MARCUS BAKER, U. S. COAST SURV., LOS ANGELES, CAL.

From (1) we have

$$\frac{a+b}{a-b}=\frac{\tan\frac{1}{2}(2\theta+2\varphi)}{\tan\frac{1}{2}(2\theta-2\varphi)}=\frac{\sin 2\theta+\sin 2\varphi}{\sin 2\theta-\sin 2\varphi}, \text{ whence } \frac{a}{b}=\frac{\sin 2\theta}{\sin 2\varphi},$$

and our two equations stand thus:—

$$b\sin 2\theta \;=\; a\sin 2\varphi, \qquad (1')$$

$$b\cos 2\theta \;=\; c-a\cos 2\varphi. \qquad (2')$$

Squaring and adding,

$$b^2 \;=\; a^2+c^2-2ac\cos 2\varphi,$$

an equation from which θ has been eliminated.

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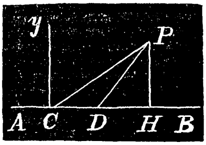
427. *By Prof. J. W. Nicholson.*—"C and D are two fixed points (1 dist. apart) on the line AB; show that the locus of a point P, moving in such a manner that $\angle PDB=n\angle PCB$, the origin being at C, is

$$\frac{y\sqrt{-1}}{x-1}=\frac{(x+y\sqrt{-1})^n-(x-y\sqrt{-1})^n}{(x+y\sqrt{-1})^n+(x-y\sqrt{-1})^n}."$$

SOLUTION BY PROF. E. B. SEITZ, KIRKSVILLE, MO.



Let $CD=1$, $CH=x$, $PH=y$, $CP=r$, $\angle PDB=\theta$ $\angle PCB=\varphi$.

Then $\tan\theta=\dfrac{y}{x-1}$, $x=r\cos\varphi$, $y=r\sin\varphi$, $\theta=n\varphi$.

$$\therefore \tan\theta=\tan n\varphi$$

$$=\frac{\sin n\varphi}{\cos n\varphi}=\frac{1}{\sqrt{-1}}\cdot\frac{(\cos\varphi+\sqrt{-1}\sin\varphi)^n-(\cos\varphi-\sqrt{-1}\sin\varphi)^n}{(\cos\varphi+\sqrt{-1}\sin\varphi)^n+(\cos\varphi-\sqrt{-1}\sin\varphi)^n}.$$

(See Chauvenet's Trigonometry, p. 128.)

Substituting the value of $\tan\theta$, and multiplying both terms of the last fraction by r^n, and then substituting x for $r\cos\varphi$, and y for $r\sin\varphi$, we have

$$\frac{y\sqrt{-1}}{x-1}=\frac{(x+y\sqrt{-1})^n-(x-y\sqrt{-1})^n}{(x+y\sqrt{-1})^n+(x-y\sqrt{-1})^n}.$$

[After deducing the given equation, Mr. Heaton remarks, "The polar equation may be found from the triangle CPD as follows:

$r\sin CPD=1.\sin CDP$, or $r\sin(n+1)\varphi=\sin n\varphi$, the polar equation.

$r\,[e^{(n+1)\phi\sqrt{-1}} - e^{-(n+1)\phi\sqrt{-1}}] = e^{n\phi\sqrt{-1}} - e^{-n\phi\sqrt{-1}}$; $\therefore$ $r\,[(\cos\varphi + \sin\varphi\sqrt{-1})^{(n+1)} - (\cos\varphi - \sin\varphi\sqrt{-1})^{(n+1)}] = (\cos\varphi + \sin\varphi\sqrt{-1})^{n} - (\cos\varphi - \sin\varphi\sqrt{-1})^{n}$; $\therefore$ $(x + y\sqrt{-1})^{(n+1)} - (x - y\sqrt{-1})^{(n+1)} = (x + y\sqrt{-1})^{n} - (x - y\sqrt{-1})^{n}$. A simpler form of the equation."]

PROBLEMS.

428. *By George Lilley, A. M.*—Two circles, of given radii, R and R_1, touch a straight line on the same side; a third circle of radius R_2 touches each of them; find the position of the circles R and R_1 and the radius of a fourth circle such that it shall touch the same straight line and each of the three given circles.

429. *By Prof. M. L. Comstock.*—A cone of given weight W, is placed with its base on an inclined plane, and supported by a weight W' which hangs by a string fastened to the vertex of the cone and passing over a pulley in the inclined plane at the same height as the vertex. Determine the conditions of equilibrium.

430. *By Prof. Milwee, Add-Ran Col. Texas.*—Given two fixed points A and B, one on each of the axes of coordinates, at the respective distances a and b from the origin; if A' and B' be taken on the axes so that $OA' + OB' = OA + OB$, find the locus of the intersection of AB' and $A'B$.

431. *By Prof J. W. Nicholson.*—Required the area of a triangle whose sides are equal to the three roots respectively of the following equation:

$$x^3 + mx^2 + nx + r = 0.$$

432. *By R. J. Adcock.*—Show that the quadrant of the ellipse equals

$$a\int_0^1 \left(\frac{1-e^2x^2}{1-x^2}\right)^{\frac{1}{2}} dx = \tfrac{1}{2}\pi\cos\theta\left[1 + (\tfrac{1}{2}\tan\theta)^2 - \frac{1}{3}\left(\frac{1.3}{2.4}\tan^2\theta\right)^2 + \frac{1}{5}\left(\frac{1.3.5}{2.4.6}\tan^3\theta\right)^2 - \frac{1}{7}\left(\frac{1.3.5.7}{2.4.6.8}\tan^4\theta\right)^2 + \text{\&c.}\right],$$

where $a =$ semi transverse axis, b semi conjugate, $e^2 = 1 - (b^2 \div a^2)$, $\tan^2\theta = e^2 \div (1 - e^2)$.

433. *By Prof. W. P. Casey.*—Given the base of a triangle, to find the locus of the vertex, when the centre of the inscribed square moves on a given conic section.

434. *By Prof. De Volson Wood.*—Find a number, the mantissa of the logarithm of which equals the number.